Numbers Basics

Number system is a very important chapter and you will get questions from this area in many competitive exams. We start with classification of numbers.

Types of numbers :

1. Natural numbers (N) = 1, 2, 3, ...

2. Whole numbers (W) = 0, 1, 2, 3, ...

3. Intezers (Z) = $-\infty$ . . . -2, -1, 0, 1, 2, 3, . . .

4. Rational numbers (Q) = The numbers of the form p/q where $q \neq 0$. Eg: $1/5$, 0.46, 0.333333

5. Irrational numbers (I) = The numbers of the form $x^{1/n} \neq$ Intezer. Also π and e also irrational numbers.

Other types of numbers:

a. Even numbers : Numbers which are exactly divisible by 2. These numbers are in the format of $2n$.

b. Odd numbers: Numbers which gives remainder 1 when divided by 2. These numbers are in the format of $2n \pm 1$.

c. Prime numbers : The numbers which are divisible by 1 and the number itself are primes. The least prime is 2.

d. Composite numbers : The numbers of which are divisible by more than 2 numbers.

The following rules related to Even and Odd numbers are important:

odd $\pm$ odd = even;

even $\pm$ even = even;

even $\pm$ odd = odd

odd $\times$ odd = odd;

even $\times$ even = even;

even $\times$ odd = even.

$odd^{(any\ number)}$ = odd

$even^{(any\ number)}$ = even

Fundamental Theorem of Arithmetic: Any intezer greater than 1 is either prime or product of primes. Writing a number as a product of primes is called prime factorization. For example, 100 can be written as $2^2 \times 5^2$

Solved Examples

1. $\frac{4}{5}$ of a number exceeds its $\frac{2}{3}$ by 8. The number is

a. 30

b. 60

c. 90

d. None of these

Correct Option : B

Explanation:

Let the number be 'a'.

$\frac{4}{5}a - \frac{2}{3}a = 8$

$\Rightarrow \frac{12a - 10a}{15} = 8$

$\Rightarrow 2a = 120 \Rightarrow a = 60$

2. If 1 is added to the denominator of fraction, the fraction becomes $\frac{1}{2}$. If 1 is added to the numerator, the fraction becomes 1. The fraction is

a. $\frac{4}{7}$

b. $\frac{5}{9}$

c. $\frac{2}{3}$

d. $\frac{10}{11}$

Correct Option: C

Explanation:

Let the required fraction be $\frac{a}{b}$. Then

$\Rightarrow \frac{a}{b+1} = \frac{1}{2} \Rightarrow 2a - b = 1$ ---- (1)

$\Rightarrow \frac{a+1}{b} = 1 \Rightarrow a - b = -1$ ------- (2)

Solving (1) & (2) we get a = 2, b=3

Fraction = $\frac{a}{b} = \frac{2}{3}$

3. The sum of two numbers is twice their difference. If one of the numbers is 10, the other number is

a. $3\frac{1}{3}$

b. 30

c. $-3\frac{1}{3}$

d. a or b

Correct Option: D

Explanation:

Let the other number be a.

Given that, $10 + a = 2(a - 10) \Rightarrow a = 30$

or if the other number is less than 10, $10 + a = 2(10 - a) \Rightarrow a = \frac{10}{3} = 3\frac{1}{3}$

4. The sum of squares of two numbers is 80 and the square of their difference is 36. The product of the two numbers is.

a. 22

b. 44

c. 58

d. 116

Correct Option: A

Explanation :

Let the numbers be x and y . Then

$$x^2 + y^2 = 80 \quad \text{and} \quad (x-y)^2 = 36$$
$$(x-y)^2 = 36 \Rightarrow x^2 + y^2 - 2xy = 36$$
$$\Rightarrow 2xy = (x^2 + y^2) - 36 = 80 - 36 = 44$$
$$\Rightarrow xy = 22$$

5. 11 times a number gives 132. The number is

a. 11

b. 12

c. 13.2

d. None

Correct Option: B

Explanation:

Let the number be 'N'
$$11 \times N = 132 \Rightarrow N = 12$$

6. If 1/5th of a number decreased by 5 is 5, then the number is

a. 25

b. 50

c. 60

d. 75

Correct Option: B

Explanation :

Let the number be a, then
$$\frac{a}{5} - 5 = 5 \Rightarrow \frac{a}{5} = 10 \Rightarrow a = 50$$

7. The difference of two numbers is 2 and the difference of their squares is 12. The sum of the number is

a. 6

b. 8

c. 10

d. 22

Correct Option: A

Explanation:

Let the numbers be a, b; $\Rightarrow$ a-b=2 and

$a^2 - b^2 = 12$

$a + b = \dfrac{a^2 - b^2}{a - b} = \dfrac{12}{2} = 6$

8. The sum of two numbers is 29 and the difference of their squares is 145. The difference between the number is

a. 13

b. 5

c. 8

d. 11

Correct Option: B

Explanation:

Let the numbers be x and y, then

$(x - y) = \dfrac{x^2 - y^2}{(x + y)}$

$= \dfrac{145}{29} = 5$

9. The difference between the squares of two consecutive numbers is 35. The numbers are

a. 14,15

b. 15,16

c. 17,18

d. 18,19

Correct Option: C

Explanation:

Let the numbers be a and (a+1)

$(a + 1)^2 - a^2 = 35$

$\Rightarrow a^2 + 2a + 1 - a^2 = 35$

$\Rightarrow 2a = 34$ or a = 17

The numbers are 17 & 18.

10. $\dfrac{3^{th}}{4}$ of $\dfrac{1^{th}}{5}$ of a number is 60. The number is

a. 300

b. 400

c. 450

d. 1200

Correct Option: B

Explanation:

Let the number be N. Then

$\dfrac{3}{4} \times \dfrac{1}{5} \times N = 60 \Rightarrow 3N = 1200 \Rightarrow N = 400$

11. 24 is divided into two parts such that 7 times the first part added to 5 times the second part gives 146. The first

part is

a. 11

b. 13

c. 16

d. 17

Correct Option: B

Explanation:

Let the first and second parts be a and 24 a, then

$7a + 5(24 - a) = 146$
$\Rightarrow 7a + 120 - 5a = 146$
$\Rightarrow 2a = 26$ or a = 13

12. The product of two numbers is 120. The sum of their squares is 289. The sum of the two numbers is :

a. 20

b. 23

c. 169

d. None

Correct Option: B

Explanation:

Let the number be x and y . Then

$(x + y)^2 = (x^2 + y^2) + 2xy = 289 + 2 \times 120$
$= 289 + 240 = 529 \Rightarrow x + y = \sqrt{529} = 23$

13. The sum of squares of two numbers is 68 and the square of their difference is 36. The product of the two

numbers is

a. 16

b. 32

c. 58

d. 104

Correct Option:a

Explanation:

Let the numbers be x and y. Then

$x^2 + y^2 = 68 \& (x - y)^2 = 36$
But $(x - y)^2 = 36 \Rightarrow x^2 + y^2 - 2xy = 36$
$\Rightarrow 68 - 2xy = 36 \Rightarrow 2xy = 32$
$\Rightarrow xy = 16$

14. The sum of seven numbers is 235. The average of the first three is 23 and that of the last three is 42. The

fourth number is

a. 40

b. 126

c. 69

d. 195

Correct Option: A

Explanation:

$(23 \times 3 + a + 42 \times 3) = 235 \Rightarrow a = 40$

15. Two numbers are such that the ratio between them is 3:5 but if each is increased by 10, the ratio between them becomes 5 : 7, the numbers are

a. 3, 5

b. 7, 9

c. 13, 22

d. 15, 25

Correct Option: D

Explanation:

Let the numbers be 3a and 5a

Then $\dfrac{3a + 10}{5a + 10} = \dfrac{5}{7}$

$\Rightarrow 7(3a + 10) = 5(5a + 10) \Rightarrow a = 5$

The numbers are 15 & 25

16. A fraction becomes 4 when 1 is added to both the numerator and denominator, and it becomes 7 when 1 is subtracted from both the numerator and denominator. The numerator of the given fraction is :

a. 2

b. 3

c. 7

d. 15

Correct Option: D

Explanation:

Let the required fraction be $\dfrac{a}{b}$

Then $\dfrac{a + 1}{b + 1} = 4 \Rightarrow a - 4b = 3$

and $\dfrac{a - 1}{b - 1} = 7 \Rightarrow a - 7b = -6$

Solving these equations we get,

a=15

b=3

17. A number exceeds 20% of itself by 40. The number is

a. 50

b. 60

c. 80

d. 320

Correct Option: A

Explanation:

Let the answer be 'a'

Then $a - \dfrac{20}{100} a = 40 \Rightarrow 5a - a = 200$

$\Rightarrow a = 50$

18. Three numbers are in the ratio 3:4:5. The sum of the largest and the smallest equals the sum of the third and 52.The smallest number is :

a. 20

b. 27

c. 39

d. 52

Correct Option: C

Explanation:

Let the numbers be 3 N, 4 N and 5 N

Then 5N+3N = 4 N +52

$\Rightarrow 4N = 52 \Rightarrow N = 13$

The smallest number =3N=3 x 13=39

19. If 16% of 40% of a number is 8, then the number is

a. 200

b. 225

c. 125

d. 320

Correct Option: C

Explanation:

Let $\dfrac{16}{100} \times \dfrac{40}{100} \times a = 8$

$a = \dfrac{8 \times 100 \times 100}{16 \times 40} = 125$

20. If 3 is added to the denominator of a fraction, it becomes $\dfrac{1}{3}$ and if 4 be added to its numerator, it becomes $\dfrac{3}{4}$; the fraction is :

a. $\dfrac{4}{9}$

b. $\dfrac{3}{20}$

c. $\dfrac{7}{24}$

d. $\dfrac{5}{12}$

Correct Option: D

Explanation:

Let the required fraction be $\dfrac{a}{b}$

Then $\dfrac{a}{b+3} = \dfrac{1}{3}$

$\Rightarrow 3a - b = 3$ and

$\dfrac{a+4}{b} = \dfrac{3}{4} \Rightarrow 4a - 3b = -16$ solving,we get

a=5, b=12; required answer = 5/12

21.Of the three numbers, the first is twice the second and is half of the third. If the average of three numbers is 56, then the smallest number is

a. 24

b. 36

c. 40

d. 48

Correct Option: A

Explanation:

Let the second number is a. Then the first number is 2a and third number is 4a.

$\dfrac{2a + a + 4a}{3} = 56 \Rightarrow 7a = 3 \times 56$ or

a= $\dfrac{3 \times 56}{7} = 24$

Smallest number is 24

22.The difference of two numbers is 8 and $\dfrac{1}{12}^{th}$ of the sum is 1. The numbers are

a. 10, 2

b. 18, 26

c. 10, 18

d. 26, 34

Correct Option: A

Explanation :

Let the numbers be a and (a+8). Then

$\dfrac{1}{12}[a + (a+8)] = 1 \Rightarrow 2a + 8 = 12$
$\Rightarrow a = 2, a + 8 = 10$

23.A number is 25 more than its $\dfrac{2}{5}$th . The number is

a. 60

b. 80

c. $\dfrac{125}{3}$

d. $\dfrac{125}{7}$

Correct Option: C

Explanation:

Let the number be N. Then

$N - 25 = \dfrac{2}{5}N \, or \, 5N - 125 = 2N$

or N=125/3

24.The sum of three numbers is 68. If the ratio between first and second be 2 : 3 and that between second and third be 5 : 3, then the second number is

a. 30

b. 20

c. 58

d. 48

Correct Option: A

Explanation:

Let the numbers be x,y,z. Then

$$\frac{x}{y} = \frac{2}{3}, \frac{y}{z} = \frac{5}{3} \Rightarrow \frac{x}{y} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$\text{and } \frac{y}{z} = \frac{5 \times 3}{3 \times 3} = \frac{15}{9}$$

$$\Rightarrow x : y : z = 10 : 15 : 9$$

25. The sum of two numbers is 100 and their difference is 37. The difference of their squares is

a. 37

b. 100

c. 63

d. 3700

Correct Option: D

Explanation:

Let the numbers be x and y

Then x+y=100 & x-y=37

$$x^2 - y^2 = (x - y)(x + y) = 100 \times 37 = 3700$$

26. What least number must be subtracted from 12702 to get number exactly 99 ?

a. 49

b. 30

c. 29

d. 31

Correct Option: B

Explanation:

99) 12702 (128

   99

  280

  198

  822

  792

   30

Required number is 30.

27. The largest number of four digits exactly divisible by 77 is

a. 9768

b. 9933

c. 9988

d. 9944

Correct Option: B

Explanation: Find the remainder when 10000 is divided by 77. Then substract that remainder from 10000. you can solve this by taking 9999 also.

28. Sum of the numbers from 1 to 20 is

a. 210

b. 110

c. 220

d. 105

Correct Option: A

Explanation: Sum of first n natural numbers = $1 + 2 + 3 + \dots n = \dfrac{n(n+1)}{2}$

Substitute n = 20.

29. Sum of even numbers between 15 and 25 is

a. 70

b. 80

c. 130

d. 100

Correct Option: D

Explanation: 16 + 18 + ...24.

Taking 2 common we get = 2 (8 + 9 + 10 +.....+ 12)

Sum of n natural numbers upto 12

8 + 9 + 10 +.....+ 12 = (1 + 2 + 3 + ...+ 12) - (1 + 2 + 3 + ...+ 7)

By applying the formula for the first n natural numbers sum $\dfrac{n(n+1)}{2}$ we get, $\dfrac{12(12+1)}{2} - \dfrac{7(7+1)}{2} = 50$

So 16 + 18 + ...24 = 2 x 50 = 100

30. How many numbers between 1000 and 5000 are exactly divisible by 225?

a. 16

b. 18

c. 19

d. 12

Correct Option: B

Explanation: First multiple of 225 after 1000 is 1125 (225 x 5) and last multiple of 225 before 5000 is 4950 (225 x 22)

Total number are $\dfrac{l-a}{d} + 1 = \dfrac{4950-1125}{225} + 1 = 18$

31. If the first 200 numbers are written down and those divisible by 2 are deleted and again those divisible by 5 are deleted, how many numbers are left out ?

a. 80

b. 150

c. 200

d. 160

Correct Option: A

Total numbers divisible by 2 = 100

Total numbers divisible by 5 = 40

But there is double counting. So we have to substract Total numbers which are divisible by both 2 and 5 i.e, 200 / 10 = 20

So Total numbers which are divisible by either 2 or 5 is 100 + 40 - 20 = 120

Number of numbers which are not divisible by any of those = 200 - 120 = 80

32. How many digits are required to write numbers between 1 to 100.

a. 196

b. 158

c. 192

d. 200

Correct Option: C

Explanation:

Single digits are from 1 to 9 = 9 digits

Doubt digits are from 10 to 99 = 90 x 2 = 180 digits

100 needs 3 digits. Total 192 digits

33. When simplified, the product :

$$[2-\frac{1}{3}][2-\frac{3}{5}][2-\frac{5}{7}] \quad . . . [2-\frac{997}{999}]$$

a. $\dfrac{5}{999}$

b. $\dfrac{1001}{999}$

c. $\dfrac{1001}{3}$

d. None of these

Correct Option: C

Explanation:

$$[2 - \frac{1}{3}][2 - \frac{3}{5}][2 - \frac{5}{7}]\dots[2 - \frac{997}{999}] \qquad = \frac{5}{3} \times \frac{7}{5} \times \frac{9}{7} \times \dots \times \frac{1001}{999}$$

$$= \frac{1001}{3}$$

34. If $2^{x-1} + 2^{x+1} = 320$, then the value of x is :

a. 4

b. 5

c. 6

d. 7

Correct Option: D

Explanation:

Try to plugin options. For x = 7 we get 320.

35. Which one of the following numbers is not a square of any natural number ?

a. 17956

b. 18225

c. 53361

d. 63592

Correct Option: D

Explanation: Square of any number ends with 0, 1, 4, 5, 6, 9 only